AN ENGINEERING METHOD OF CALCULATING A JET IN A FLUIDIZED BED

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An engineering method is described for calculating a gas jet in a fluidized bed on the basis of a solution for the kinematics of the jet.

An approximate solution has been obtained [1, 2] for the kinematics of a jet in a fluidized bed, on the assumption that the pole of the basic section of the jet is placed in the initial section and that one neglects the influence of the entrained flow on the development of the jet; then we get for the increase in the total thickness in the boundary layer in the basic section and the change in the velocity at the axis that

$$b=\left(C_1\frac{U_m-U_b}{U_m+U_b}+C_2\right)X, \tag{1}$$

$$U_m=\frac{U_0 r_0}{0.366\,b}\sqrt{\frac{\rho_g}{\rho_{av}}}. \tag{2}$$

The half-width of the boundary layer at the point where the gas jet closes ($U_m = U_b$) is determined by

$$b_{X_j}=C_2 X_j \tag{3}$$

while the current half-width of the gas jet and the extent is defined by

$$b_j=C_1\frac{U_m-U_b}{U_m+U_b}X, \tag{4}$$

$$X_j=\frac{U_0 r_0}{0.366\,U_b C_2}\sqrt{\frac{\rho_g}{\rho_m}}. \tag{5}$$

Equations (1)-(5) have been derived on the assumption that there is an affine velocity distribution in the boundary layer [1, 2], and for the basic solution one assumes the universal profile described by Schlichting's equation [5, 7]:

$$\frac{U}{U_m}=\left[1-\left(\frac{y}{b}\right)^{1.5}\right]^2 \quad \text{for} \quad \frac{y_c}{b}=0.44. \tag{6}$$

The assumptions made in solving and analyzing this problem have been confirmed by experiment [1-3, 6] over a wide range of parameters for the layer and the jet. It has been found [3] that the Schlichting profile for a jet in a fluidized bed is only a satisfactory approximation to two different laws of variation of the velocity over the zones of the boundary layer; it is therefore more accurate to determine the velocity profile by zones: in the gas zone from

$$\frac{U-U_b}{U_m-U_b}=1-\left[1-\left(1-\frac{y}{b_j}\right)^{1,5}\right]^2 \quad \text{for} \quad \frac{y_b}{b_j}=0.56, \tag{7}$$

and in the zone of gas + solid particles by a graphical analytical method [3].

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The approximate solution shows that (1) is derived on the assumption of constant increase in the jet thickness in the two zones, which means that the expression for the velocity at the closure point may be put in the form

$$U_b = \frac{U_0 r_0}{0.366\, C_1 X_j}. \tag{8}$$

Comparison of (5) and (8) shows that there is the following relation between the jet coefficients C_1 and C_2:

$$C_1 = C_2 \sqrt{\frac{\rho'_m}{\rho_g}} \tag{9}$$

This relation has been confirmed by experiment to 15% [1].

One therefore needs to know one experimental coefficient in order to calculate a jet in a fluidized bed in the same way as one calculates for an enclosed jet [5].

Equation (2) contains the mean density ρ_{av}, which may be determined by averaging the mean densities for the two zones [1]. For each section of the boundary layer we can express the momentum via the mean density and represent it as the sum of the amounts of momentum in each zone; then for an axially symmetric jet we have

$$b^2 \rho_{av} U_{av}^2 = b_j^2 \rho_g U_{av1}^2 + (b^2 - b_j^2)\, \rho'_m U_{av2}^2. \tag{10}$$

The solution to (10) for ρ_{av} is

$$\rho_{av} = \rho_g \frac{b_j^2}{b^2} \cdot \frac{U_{av1}^2}{U_{av}^2} + \rho'_m \frac{b^2 - b_j^2}{b^2} \frac{U_{av2}^2}{U_{av}^2}. \tag{11}$$

The mean density in the boundary layer varies along the length in accordance with the gas density ρ_g at the edge of the nozzle up to the mean density in the mixed zone ρ'_m at the point $X = X_j$. In this case, to satisfy (10) at any point along the jet it is necessary for the sum of the coefficients to ρ_g and ρ'_m to sum to unity in (11), i.e.,

$$\frac{b_j^2}{b^2} \cdot \frac{U_{av1}^2}{U_{av}^2} + \left(1 - \frac{b_j^2}{b^2}\right) \frac{U_{av2}^2}{U_{av}^2} = 1. \tag{12}$$

It follows from (12) that

$$U_{av}^2 = U_{av1}^2 \frac{b_j^2}{b^2} + U_{av2}^2 \left(1 - \frac{b_j^2}{b^2}\right). \tag{13}$$

Assuming that

$$U_{av_1} = U_b + 0.84\,(U_m - U_b), \tag{14}$$

and

$$U_{av_2} = \frac{U_b}{2}, \tag{15}$$

and expressing b_j/b in accordance with (1) and (4) as

$$\frac{b_j}{b} = \frac{C_1 A}{C_1 A + C_2}, \tag{16}$$

where

$$A = \frac{U_m - U_b}{U_m + U_b},$$

we obtain, after substitution of (13)-(16) into (11) and certain transformations, the equation for the mean density in the boundary layer of the jet in the main section:

$$\rho_{av} = \rho_g \frac{C_1^2 A^2 (0.84\, U_m + 0.16\, U_b)^2}{C_1^2 A^2 (0.84\, U_m + 0.16\, U_b)^2 + \frac{U_b^2}{4}(2C_1C_2A + C_2^2)} + \rho_m' \frac{\frac{U_b^2}{4}(2C_1C_2A + C_2^2)}{C_1^2 A^2 (0.84\, U_m + 0.16\, U_b)^2 + \frac{U_b^2}{4}\left(2C_1C_2A + C_2^2\right)}. \tag{17}$$

If ρ_g is much less than ρ_m, as follows from (1)-(5), (9), and (17), it is inevitable that a discontinuity occurs in the jet [1, 4]. This is confirmed by experiment [3, 6], which shows that the frequency of jet disruption does not remain constant and is dependent on the flow conditions; this frequency f_m may be 6-48 Hz or more in the jet condition, and the flow of the jet may be considered as quasistationary, with constant maximal parameters in the fluidized bed, which can be calculated via (1)-(5), (7)-(9), and (17). The parametric criterion X_j/H_p [3] defines the existence of the jet condition and, as a special case, the condition for local existence of a fountain.

We can use (1)-(9) and (17) for kinematic calculation of a gas jet if we are given either C_1 or C_2, as well as ρ_m' and U_b. It has been found by experiment for local fountain conditions [3] in semiinfinite (horizontal and vertical) gas jets that U_b is constant along the jet and is independent of the velocity and direction of discharge, the nozzle diameter, and the fluidization number. To determine U_b we can use a formula [8] as corrected via the factor $\eta = 0.80$, which has been derived by experiment for particles 1-5 mm in size:

$$\mathrm{Re}_b = \frac{0.80\,\mathrm{Ar}}{18 + 0.61\sqrt{\mathrm{Ar}}} \tag{18}$$
$$480 \leqslant \mathrm{Re}_b \leqslant 2300;\ 19.8\cdot 10^4 \leqslant \mathrm{Ar} \leqslant 313\cdot 10^4.$$

The experimental constant C_1 reflects the effects of various factors not explicitly incorporated, in particular, the initial microstructure of the turbulent flow, etc; this was determined by the above method using data only on the change of velocity along the axis. The jet flow speed in the local-fountain state varied from 12 to 310 m/sec. The jets emerged from fittings of diameters 2, 3, 4, 6, 8, and 10 mm which were shaped in accordance with the equation of [9]; the initial calculated equation for C_1 is

$$C_1 = \frac{U_0 r_0}{0.366\, U_b X_j}. \tag{19}$$

The U_0 and r_0 of (19) are known from conditions of the problem, while X_j is determined from experiment [3], and U_b is calculated from (18) (for the given range in the Re_b and Ar), or else it is determined by experiment. Equation (19) gives reliable C_1, because the velocity profile derived from (7) agrees well with that over the full width of the boundary layer at various parts in the jet as calculated from (7); the spread in the experimental points does not exceed 7.7%.

Measurements on a jet in a monodisperse fluidized bed showed that C_1 is 3-4 times that for a jet entering a gas, which agrees with experiment [1], in which C_1 was determined from the velocity profile of the jet. Also, C_1 is not affected by the jet-flow speed, initial diameter, apparatus diameter, and particle density within the ranges used: $5600 \le \mathrm{Re}_0 \le 310{,}000$; $62.3 \le \mathrm{Re}_m \le 512$; $19.8\cdot 10^4 \le \mathrm{Ar} \le 313\cdot 10^4$; $0.016 \le d_0/D \le 0.08$. Independence of C_1 from the flow speed and nozzle diameter (over a wide range in Re_0) is characteristic also for free jets entering gas [5]. However, C_1 is dependent on the particle diameter, the fluidization number (Fig. 1), and the degree of homogenizing action of the jet on the fluidized bed, which is determined via the uniformity parameter

$$n = \frac{Q_p - Q_c}{Q_c} = \frac{\Delta Q}{Q_c}, \tag{20}$$

the ratio of the gas in the layer in excess of that necessary for start of fluidization, ΔQ, to the initial jet mass Q_c.

The jet homogenizes the fluidized bed (Fig. 2) in jet flow and local fountain cases, while in bubble flow there is no such effect [3, 6]. In the local-fountain case for $n^* \lessapprox 1.5$-2.0 (in the range of fluidization

*Accurate to 16%.

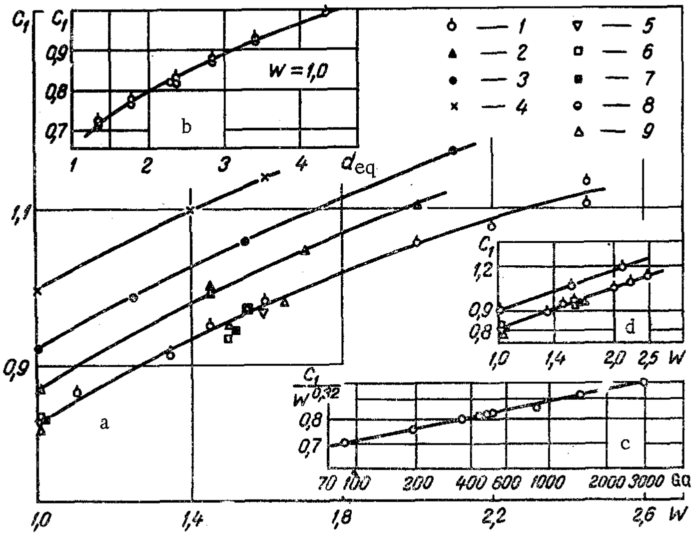


Fig. 1. Effect of the parameters of a fluidized bed on the value of the coefficient C_1 (a, b) and on the generalization of the experimental data by C_1 (c, d). Alumosilicate catalyst ($\rho_s = 115\ \mathrm{kg \cdot sec^2/m^4}$): 1) fraction 2-2.5 mm; 2) 2.5-3.0; 3) 3-4; 4) 4-5. Urea-ammophos ($\rho_s = 155$): 5) fraction 1-4 mm ($d_{eq} = 2.0$). Granulated nitrogen–phosphorus–potassium fertilizer ($\rho_s = 180$): 6) fraction 2-2.5 mm; 7) 1-4 ($d_{eq} = 2.1$ mm). Glass spheres ($\rho_s = 280$): 8) fraction 1.25-1.40 mm; 9) 2-2.5.

numbers from 1.0 to 2.0) the fluidized bed acquires properties characteristic of the uniform state [6]. Also, C_1 is dependent on the particle diameter alone, while the effects of the fluidization number on C_1 are eliminated.

If $n \gtrsim 2.3$-2.8 (in the range $1.0 < W \le 2.5$) we find degeneration of the homogenization effect, i.e., degeneration of the effects of n on C_1; the state of the layer is nonuniform, and C_1 is dependent on the particle diameter and on the fluidization number. In the narrow range 1.5-$2.0 < n < 2.3$-2.8, C_1 is a function of particle diameter, fluidization number, and parameter n. Here, the larger right limit to n corresponds to a smaller value of the fluidization number.

Then the effects of the parameters on C_1 in the general case of a nonuniform bed (in the range $1.0 \le W \le 2.5$ and $n \gtrsim 2.3$-2.8) indicates there is a relationship of the form

$$C_1 = C_1(d_{eq}, W). \tag{21}$$

As $W = w_m/w_0$, $w_0 = w_0(d_{eq}, \rho_s, \rho_g, \nu)$, from (21) we get

$$C_1 = C_1(w_m, d_{eq}, \rho_s, \rho_g, \nu). \tag{22}$$

The detailed form of the relationship may be deduced by the method of dimensions [10, 11]; of the six independent variables in (22), only three independent units of measurement (m, kg, sec) are available. The π-theorem [11] indicates that the relations between the six variables may be represented in this case as a product of three dimensionless groupings:

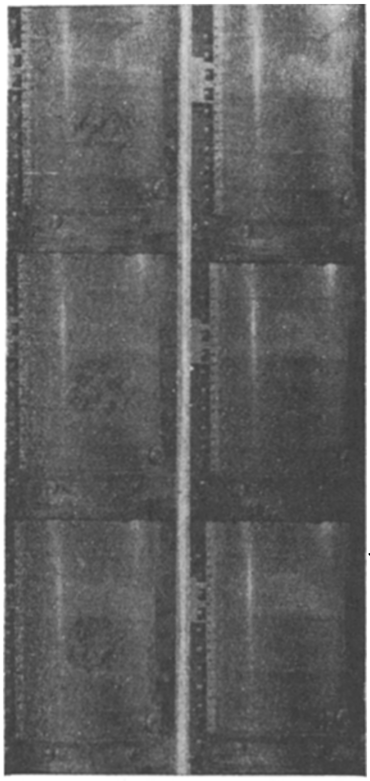
$$C_1 = A\mathrm{Re}_m^{a_1}\mathrm{Fr}^{a_2}\left(\frac{\rho_s}{\rho_g}\right)^{a_3}. \tag{23}$$

In (23) we replace Fr by the derived Galileo number [10] and exclude the grouping ρ_s/ρ_g from the parameters to get

$$C_1 = A_1\,\mathrm{Re}_m^{b_1}\mathrm{Ar}^{b_2}. \tag{24}$$

Equation (24) may be put in the form

$$C_1 = A_2\,\mathrm{Ga}^{b_3}W^{b_4}. \tag{25}$$



a b

Fig. 2. Qualitative changes in the state of the layer for jet flow in the local-spouting mode: a) material of layer, alumosilicate catalyst, fraction 2-2.5 mm; $H_0 = 125$ mm; $D = 125$ mm; $W = 1.2$ (no jet); b) the same, but with jet injection along the axis of the apparatus; $r_0 = 5$ mm; $U_0 = 30$ m/sec; $n = 0.85$; $X_f/H_p = 0.66$. Filming at 24 frames /sec.

We processed results from 170 runs with 8500 local-velocity measurements to use (25) with successive elimination of the criteria (Fig. 1) to get the following equation for C_1 to ±4.8% in a nonuniform fluidized bed:

$$C_1 = 0.46\,\mathrm{Ga}^{0.095}\,W^{0.32},$$
$$88 \leqslant \mathrm{Ga} \leqslant 3160;\ 1.0 \leqslant W \leqslant 2.5; \tag{26}$$
$$2.3 - 2.8 \leqslant n \leqslant 22.$$

For the uniform state ($1 \leq W \leqslant 2$; $n \leq 1.5$-2.0) and for the critical initial fluidization velocity ($W = 10$), the equation for C_1 is derived from (26) as a particular case with $W = 1.0$ and is

$$C_1 = 0.46\,\mathrm{Ga}^{0.095}. \tag{27}$$

We processed the experimental results via (24) to get a calculated equation for C_1 that contains only the initial parameters of the bed:

$$C_1 = 0.81\,\mathrm{Ar}^{-0.115}\,\mathrm{Re}_m^{0.32},$$
$$19.8\cdot 10^4 \leqslant \mathrm{Ar} \leqslant 313\cdot 10^4. \tag{28}$$

The range in Re_m for each material is defined by $1.0 \leq W \leq 2.5$; the range in Re_m in the experiments was 62.3 to 512. In the narrow range $(1.5\text{-}2.0) < n < (2.3\text{-}2.8)$ one can determine C_1 as the arithmetic mean of the values obtained from (26) and (27).

We deduced ρ'_m via experimental results for the velocity at the axis as used in (1), (2), (9), (17), (18), (20), (26)-(28) by successive approximation to the point where (2) became an identity to ±3-4%.

We found that ρ'_m was constant along the length of this jet and was dependent only on the diameter and density of the particles; then for $W = 1.0$ we get (see Fig. 3) a linear relation to the equivalent diameter of the particles in the layer, which can be expressed as

$$\frac{\rho_m}{\rho_m^0} = 0.34 - 0.035\,d_{eq}. \tag{29}$$

Equation (29) has been tested for the ranges $1.3 \leq d_{eq}$, mm ≤ 4.35; $115 \leq \rho_s$, $\mathrm{kg\cdot sec^2/m^4} \leq 280$; $0.38 \leq \varepsilon \leq 0.56$.

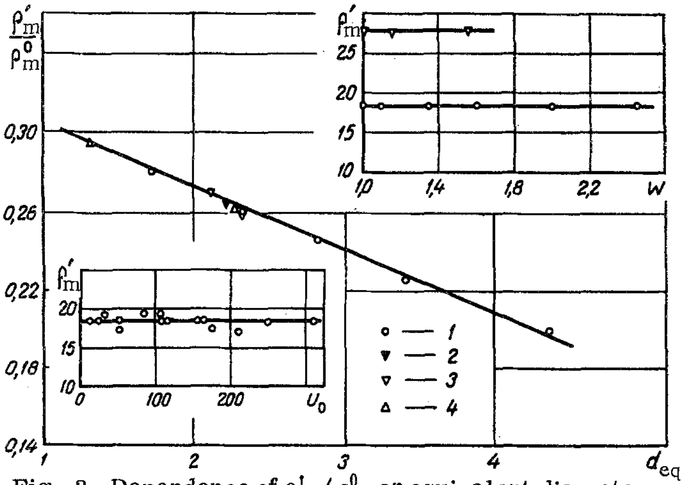


Fig. 3. Dependence of ρ'_m/ρ^0_m on equivalent diameter of bed particles: 1) alumosilicate; 2) urea-ammophos (ρ_s = 155); 3) granulated nitrogen–phosphorus –potassium fertilizer (ρ_s = 180); 4) glass spheres (ρ_s = 280). ρ'_m, kg · sec^2/m^4; d_{eq}, mm.

This method therefore requires one to know the density and fractional composition of the layer material, the fluidization number, and of the jet and layer, together with the nozzle geometry. The method provides a basis for calculating the working parameters of the bed H_p and the jet (U_0 or d_0) in a set condition; it also enables one to determine for given parameters of jet and bed the geometry of the jet, the velocity change along the axis, and the circulation speed of the particles in the jet. The error of calculation does not exceed ±11%.

Also, H_p and U_0 or d_0 can be derived for given conditions via (1)-(3), (12)-(14) via [3] and (8). Here one first of all determines the speed at the boundary of the gas jet, U_b, from (18) and C_1 from (26)-(28). To calculate for a circular jet and the particle circulation one first determines the n of (20), the U_b of (18), the ρ'_m of (29), and the jet coefficients C_1 of (26)–(28) and C_2 of (9). Then, one uses a series of values U_{m_i} from U_0 to U_b to determine for each case the ρ_{av} of (17), the b of (2), the X of (1), and the b_j of (4). The results are plotted as curves for ($b = b(X)$; $b_j = b_j(X)$) with allowance for curvature of a horizontal jet, as in (10) and (11) [3].

The method for an annular jet is entirely as above for a circular jet flowing in a layer with a velocity equivalent as regards momentum, as in (15) [3].

The curve for the axial velocity $U_m(X)$ in the main part is derived from (1), (2), (9), (17), (18), (20), (26)-(28), while in the transition part it is deduced by a graphical method with smooth connection of the end of the initial part given by (4) [3] and the start of the main part of (5) [3] for the curve for $U_m(X)$. The distance X_c is first calculated via the above method for $U_{m_i} = U_0/2$.

Velocity profile in the main part is calculated via (7), while in the mixed region it is calculated by the graphical method of [3].

It has been found that this method can be used for a jet in a polydisperse fluidized bed, provided the equivalent particle diameter is used and if the mixture contains up to 70% of fractions differing by diameter by a factor of 3; it can also be extended with an error of up to 20% to the bubble condition in the following ranges of X_j/H_p [3]:

for the vertical jet:

$$0.26 < X_j/H_p < 0.60, \tag{30}$$

for the horizontal jet:

$$\begin{gathered} 0.26 < X_j/H_p < 0.80, \\ 0.26 \leqslant X_j/H_p \leqslant 2.4;\ 1{,}3 \leqslant d_{eq} \leqslant 4.35;\ 115 \leqslant \rho_s,\ \mathrm{kg \cdot sec^2/m^4} \leqslant 280; \\ 1 \leqslant W \leqslant 2.5;\ 2 \cdot U_b \leqslant U_0 \leqslant 310\ \mathrm{m/sec}. \end{gathered} \tag{31}$$

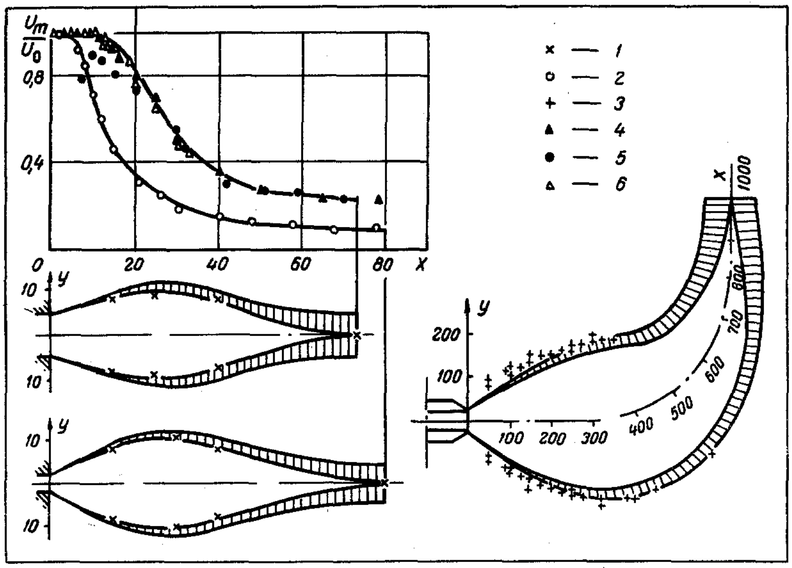


Fig. 4. Comparison of calculated curves of variation of axial velocity and outlines of gas flame with experimental data. Continuous curves, calculated results: 1) dynamic flame boundary (according to experiment). Granulated nitrogen–phosphorus–potassium fertilizer, fraction 1–4 mm (d_{eq} = 2.1 mm, ρ_s = 180 kg · sec²/m⁴); 2) W = 1.0; d_0 = 4 mm; U_0 = 83.5 m/sec; $T_0 = T_b$ = 28 ± 3°C; 3) experimental thermal boundary of flame in pilot granulator with pneumatic-sprayer parameters: 240 kg · pulp/h; alumosilicate catalyst fraction 2-2.5 mm (d_{eq} = 2.34 mm; ρ_s = 115 kg · sec²/m⁴, W = 1.0); 4) d_0 = 10 mm; U_0 = 26.1 m/sec (vertical jet); 5) d_0/d = 10/7; U_{OK} = 36.6 m/sec; U_{EKV} = 26.1 m/sec (annular jet); 6) d_0 = 10 mm; U_0 = 26.1 m/sec (horizontal jet).

The method has been tested in the laboratory and on plant (Fig. 4) for jet speeds from $2U_b$ to 310 m/sec, nozzle diameters of 2-70 mm, particle densities from 115 to 280 kg · sec²/m⁴, fluidization numbers from 1 to 2.5, and d_0/D from 0.016 to 0.08.

Granulators and other such units may use nonisothermal jets, so we examined the scope for error in the jet shape, which showed that there was ±4-11.5% error in the jet width and ±15-19.2% error in the length for $U_b/U_0 > 0.2$ on account of the nonisothermal conditions when $0.5 \leqslant \Theta = T_0/T_b \leqslant 2.0$, which are acceptable limits of error. A special study is needed to examine the effects of nonisothermal hot jets over a wider range in Θ in propagation in fluidized beds.

NOTATION

U_0	is the initial velocity of round jet outflow, m/sec;
U, U_m	are the velocity along radius (within given section) and along jet axis;
U_b	is the velocity at the gas torch boundary;
$U_{av}, U_{av_1}, U_{av_2}$	are the mean velocities in jet boundary layer section, of gas torch and "gas–solid particles" region;
y, b, b_j	are the current radius (ordinate), half-width of boundary layer of jet and of gas torch (at the same section);
C_1, C_2	are the experimental coefficients of jet;
$\rho_g, \rho_s, \rho'_m, \rho_{av}, \rho^0_m$	are the density of fluidizing agent (jet), of solid particles, gas–solid particles region, section of jet and fluidized bed at W = 1.0;
ν	is the kinematic viscosity of fluidizing agent (jet);
$d_0 (r_0)$	is the diameter (radius) of round nozzle;
X, X_c, X_j	are the distance along axis from nozzle edge to given section of jet (abscissa) to the point where velocity is $U_0/2$, and maximum length of gas torch;
f_m	is the frequency of jet generation cycles;

d_{eq}	is the equivalent diameter of layer particles, mm;
w_m, w_0	are the operating and critical initial rates of fluidization;
W	is the fluidization number;
ε	is the mean velocity of layer;
H_p, H_0	are the operating and stationary height of layer over nozzle (vertical jet) and over nozzle axis (horizontal jet);
T_0, T_b	are the initial temperatures of the jet and the fluidization layer;
n	is the uniformity parameter;
Q_c	is the initial mass of jets reduced to conditions in layer;
Q_k, Q_p	are the air-flow rate for fluidization at critical initial and operating rates of fluidization;
P_{fa}	is the pressure of pulverizing agent on sprayer;
$Re_m = w_m d_{eq}/\nu$; $Re_b = U_b d_{eq}/\nu$; $Re_0 = U_0 d_0/\nu$; $Ga = gd_{eq}^3/\nu^2$; $Ar = (gd_{eq}^3/\nu^2) \cdot ((\rho_s - \rho_g)/\rho_g)$	are the Reynolds, Galilean, and Archimedean numbers.

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